1. The dimension of the space spanned by the vectors $(-1, 0, 1, 2)$, $(-2, -1, 0, 1), (-3, 2, 0, 1)$ and $(0, 0, -1, 1)$ is

   A. 1
   B. 2
   C. 3
   D. 4.

2. How many onto functions are there from a set A with $m > 2$ elements to a set B with 2 elements?

   A. 2^m
   B. $2^m - 1$
   C. $2^{m-1} - 2$
   D. $2^m - 2$.

3. The function $f : \mathbb{R}^2_+ \to \mathbb{R}$ given by $f(x, y) = xy$ is

   A. quasiconcave and concave
   B. concave but not quasiconcave
   C. quasiconcave but not concave
   D. none of the above.

4. The function $f : \mathbb{R}^2_+ \to \mathbb{R}$ given by $f(x, y) = xy$ is

   A. homogeneous of degree 0
   B. homogeneous of degree 1
   C. homogeneous of degree 2
   D. not homothetic.

5. You have n observations on rainfall in centimeters (cm) at a certain location, denoted by x, and you calculate the standard deviation, variance, and coefficient of variation (CV). Now, if instead, you were given the same observations measured in millimeters (mm), then

A. the standard deviation and CV would increase by a factor of 10, and the variance by a factor of 100

B. the standard deviation would increase by a factor of 10, the variance by a factor of 100, and the CV would be unchanged

C. the standard deviation would increase by a factor of 10, and the variance and CV by a factor of 100

D. none of the above.

6. You have n observations on rainfall in centimeters (cm) at two locations, denoted by x and y respectively, and you calculate the covariance, correlation coefficient r, and the slope coefficient b of the regression of y on x. Now, if instead, you were given the same observations measured in millimeters (mm), then

A. the covariance would increase by a factor of 10, b by a factor of 100, and r would be unchanged

B. the covariance and b would increase by a factor of 100, and r would be unchanged

C. the covariance would increase by a factor of 100, and b and r would be unchanged

D. none of the above.

7. Let $0 < p < 100$. Any solution (x^*, y^*) of the constrained maximization problem

$$\max_{x,y} \left(\frac{-1}{x} + y \right)$$

subject to

$$\begin{aligned} px + y &\leq 10, \\ x, y &\geq 0, \end{aligned}$$

must satisfy

A. $y^* = 10 - p$

B. $x^* = 10/p$

C. $x^* = 1/\sqrt{p}$

D. none of the above.

8. Suppose the matrix equation $Ax = b$ has no solution, where A is a 3×3 non-zero matrix of real numbers and b is an 3×1 vector of real numbers. Then,

   A. The set of vectors x for which $Ax = 0$ is a plane.

   B. The set of vectors x for which $Ax = 0$ is a line.

   C. The rank of A is 3.

   D. $Ax = 0$ has a non-zero solution.

9. k people get off a plane and walk into a hall where they are assigned to at most n queues. The number of ways in which this can be done is

   A. C_k^n

   B. P_k^n

   C. $n^k k!$

   D. $n(n+1)\ldots(n+k-1)$.

10. If $Pr(A) = Pr(B) = p$, then $Pr(A \cap B)$ must be

    A. greater than p^2

    B. equal to p^2

    C. less than or equal to p^2

    D. none of the above.

11. If $Pr(A^c) = \alpha$ and $Pr(B^c) = \beta$, (where A^c denotes the event 'not A'), then $Pr(A \cap B)$ must be

    A. $1 - \alpha\beta$,

    B. $(1-\alpha)(1-\beta)$

    C. greater than or equal to $1 - \alpha - \beta$

    D. none of the above.

12. The density function of a normal distribution with mean μ and standard deviation σ has inflection points at

    A. μ

    B. $\mu - \sigma, \mu + \sigma$

C. $\mu - 2\sigma, \mu + 2\sigma$

D. nowhere.

13. In how many ways can five objects be placed in a row if two of them cannot be placed next to each other?

    A. 36

    B. 60

    C. 72

    D. 24.

14. Suppose $x = 0$ is the only solution to the matrix equation $Ax = 0$ where A is $m \times n$, x is $n \times 1$, and 0 is $m \times 1$. Then, of the two statements (i) The rank of A is n, and (ii) $m \geq n$,

    A. Only (i) must be true

    B. Only (ii) must be true

    C. Both (i) and (ii) must be true

    D. Neither (i) nor (ii) has to be true.

15. Mr A is selling raffle tickets which cost 1 rupee per ticket. In the queue for tickets, there are n people. One of them has only a 2-rupee coin while all the rest have 1-rupee coins. Each person in the queue wants to buy exactly one ticket and each arrangement in the queue is equally likely to occur. Initially, Mr A has no coins and enough tickets for everyone in the queue. He stops selling tickets as soon as he is unable to give the required change. The probability that he can sell tickets to all people in the queue is:

    A. $\frac{n-2}{n}$

    B. $\frac{1}{n}$

    C. $\frac{n-1}{n}$.

    D. $\frac{n-1}{n+1}$.

16. Out of 800 families with five children each, how many families would you expect to have either 2 or 3 boys? Assume equal probabilities for boys and girls.

    A. 400

    B. 450

C. 500

D. 550

17. The function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0. \end{cases}$$

is

A. concave

B. convex

C. neither concave nor convex

D. both concave and convex

18. As $n \to \infty$, the sequence $\{\frac{n^2+1}{2n^2+3}\}$

A. diverges

B. converges to $1/3$

C. converges to $1/2$

D. neither converges nor diverges.

19. The function $x^{1/3}$ is

A. differentiable at $x = 0$

B. continuous at $x = 0$

C. concave

D. none of the above.

20. The function $\sin(\log x)$, where $x > 0$

A. is increasing

B. is bounded and converges to a real number as $x \to \infty$

C. is bounded but does not converge as $x \to \infty$

D. none of the above.

21. For any two functions $f_1 : [0, 1] \to \mathbb{R}$ and $f_2 : [0, 1] \to \mathbb{R}$, define the function $g : [0, 1] \to \mathbb{R}$ as $g(x) = \max(f_1(x), f_2(x))$ for all $x \in [0, 1]$.

A. If f_1 and f_2 are linear, then g is linear

B. If f_1 and f_2 are differentiable, then g is differentiable

C. If f_1 and f_2 are convex, then g is convex

D. None of the above

22. Let $f : \mathbb{R} \to \mathbb{R}$ be the function

$$f(x) = x^3 - 3x \ \forall \ x \in \mathbb{R}.$$

Find the maximum value of $f(x)$ on the set of real numbers x satisfying $x^4 + 36 \leq 13x^2$.

A. 18

B. -2

C. 2

D. 52

23. A monkey is sitting on 0 on the real line in period 0. In every period $t \in \{0, 1, 2, \ldots\}$, it moves 1 to the right with probability p and 1 to the left with probability $1 - p$, where $p \in [\frac{1}{2}, 1]$. Let π_k denote the probability that the monkey will reach positive integer k in some period $t > 0$. The value of π_k for any positive integer k is

A. p^k

B. 1

C. $\frac{p^k}{(1-p)^k}$

D. $\frac{p}{k}$.

24. Refer to the previous question. Suppose $p = \frac{1}{2}$ and π_k now denotes the probability that the monkey will reach any integer k in some period $t > 0$. The value of π_0 is

A. 0

B. $\frac{1}{2^k}$

C. $\frac{1}{2}$

D. 1

25. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function with $f'(x) > 0$ for all $x \in \mathbb{R}$ and satisfying the property

$$\lim_{x \to -\infty} f(x) \geq 0.$$

Which of the following must be true?

A. $f(1) < 0$

B. $f(1) > 0$

C. $f(1) = 0$

D. None of the above

26. For what values of x is

$$x^2 - 3x - 2 < 10 - 2x$$

A. $4 < x < 9$

B. $x < 0$

C. $-3 < x < 4$

D. None of the above

27. $\int_e^{e^2} \frac{1}{x(\log x)^3} dx =$

A. $3/8$

B. $5/8$

C. $6/5$

D. $-4/5$

28. The solution of the system of equations

$$\begin{aligned} x - 2y + z &= 7 \\ 2x - y + 4z &= 17 \\ 3x - 2y + 2z &= 14 \end{aligned}$$

is

A. $x = 4, y = -1, z = 3$

B. $x = 2, y = 4, z = 3$

C. $x = 2, y = -1, z = 5$

D. none of the above.

29. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a twice-differentiable function with non-zero second partial derivatives. Suppose that for every $x \in \mathbb{R}$, there is a unique value of y, say $y^*(x)$, that solves the problem

$$\max_{y \in \mathbb{R}} f(x, y).$$

Then y^* is increasing in x if

A. f is strictly concave

B. f is strictly convex

C. $\frac{\partial^2 f}{\partial x \partial y} > 0$

D. $\frac{\partial^2 f}{\partial x \partial y} < 0$.

30.
$$\int 3^{\sqrt{2x+1}} dx =$$

A.
$$\frac{3^{\sqrt{2x+1}}}{\ln 3} + \frac{\sqrt{2x+1}}{\ln 3} + c$$

B.
$$\frac{3^{\sqrt{2x+1}}\sqrt{2x+1}}{\ln 3} - \frac{3^{\sqrt{2x+1}}}{(\ln 3)^2} + c$$

C.
$$\frac{3^{\sqrt{2x+1}}\sqrt{2x+1}}{(\ln 3)^2} - \frac{3^{\sqrt{2x+1}}}{\ln 3} + c$$

D. none of the above.